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Critical formation of trapped surfaces in collisions of non-expanding gravitational shock waves in de Sitter space-time

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Abstract: We study the formation of marginally trapped surfaces in the head-on collision of two shock waves in de Sitter space-time as a function of the cosmological constant and the shock wave energy. We search for a marginally trapped surface on the past light cone of the collision plane. For space-time dimensions $D \geq 3$, there exists a critical value of the shock wave energy above which there is no trapped surface of this type. For $D > 3$, the critical value of the shock wave energy depends on the de Sitter radius, and there is no this type trapped surface formation for a large cosmological constant. For $D = 3$, the critical value of the shock wave energy is independent of the cosmological constant. At the critical point, the trapped surface is finite. Below the critical energy value, the area of the trapped surface depends on the cosmological constant and the shock wave energy.

Keywords: Classical Theories of Gravity, Black Holes

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Contents

1 Introduction

It is a well-established experimental fact that our current universe is expanding with constant acceleration, which is well described by the extremely small cosmological constant $\Lambda = 10^{-47}\,\mathrm{GeV}^4$, $\Omega_\Lambda = 0.726 \pm 0.015$ [1]. There is a common opinion that two ultrarelativistic point particles in the Minkowski space-time can produce black holes [2–4]. This longstanding question has a purely theoretical meaning as well as an astrophysical one [5]. In the framework of TeV-gravity [6], black hole production in collisions of particles with the center-of-mass energy of a few TeV and their experimental signatures [7] became the subject of numerous investigations [8–12]. We also note a discussion of the possible production of wormholes and other more exotic objects at the LHC [13–15] (see [16] for a consideration of wormholes in astrophysics).

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Our main aim in this paper is to determine if the presence of a positive cosmological constant can influence the processes of black hole formation in a collision of two ultrarelativistic point particles. Intuitively, it is almost clear that a small cosmological constant cannot have a detectible influence. Meanwhile, a positive cosmological constant generates a repulsion of matter, and a critical value of the cosmological constant Λ should exist above which black hole formation is suppressed. Finding this critical value is worthwhile. We note that under the assumption of an asymptotically flat space-time, the presence of a trapped surface usually guarantees the existence of the event horizon [17–20].

We study the formation of marginally trapped surfaces in a head-on collision of two shock waves in the de Sitter space-time as a function of the cosmological constant and the square of the shock wave energy. For $D = 4$, we show explicitly that there exists a critical value for the ratio of the shock wave energy $\bar{p}$ and the cosmological radius

$$\frac{\bar{p}}{a} = \frac{1}{4G} \tag{1.1}$$

above which there is no solution of the trapped surface equation. Here G is the gravitational constant and the cosmological radius a is related to the cosmological constant by $a^2 = 3/\Lambda$. Similar results were found for $D > 4$. There is a critical value for the shock wave energy above which there are no solutions of the trapped surface equation. This value depends on the cosmological radius. At the critical point, the trapped surface is finite.

This effect is similar to the emergence of critical behavior with respect to the wave width in the transverse space when a marginally trapped surface is formed in the head-on collision of two shock waves in the Minkowski space-time [21, 22]. This effect depends on the number of dimensions and is a Choptuik-like critical effect, i.e., it is similar to the critical Choptuik behavior in gravitational collapse [23] (see [24] and the references therein).

The formation of a marginally trapped surface in the collision of gravitational shock waves in AdS_D was recently studied in [25, 26]. These studies are aimed at better understanding the entropy production in relativistic heavy ion collisions due to black hole production in a dual description. Despite the absence of a holographic dual description of QCD, describing the colliding heavy ions in terms of colliding gravitational shock waves in the anti-de Sitter space-time was suggested [27, 28]. Black hole formation in collisions of the dual of the nuclei in the bulk is interpreted as formation of a quark-gluon plasma [29–31]. In AdS, a dimension-dependent critical behavior with respect to the wave width in the transverse space in the formation of a marginally trapped surface in the head-on collision of two shock waves was recently found [21]. For $D = 4$ and $D = 5$, there exists a critical value of this width above which the trapped surface never forms. We note that in the AdS space-time, the obtained results are qualitatively the same as those obtained in the flat space-time.

This paper is organized as follows. We start with the setup and recall some basic facts about the generalization of the Aichelburg-Sexl shock wave geometry [32–34] to non-expanding shock waves propagating in D-dimensional space-times with the cosmological constant [35–43]. In section 3, we calculate the critical value of cosmological constant depending on the shock wave energy below which the trapped surface occurs and give the area of the trapped surface.

2 Setup

Our main aim in this section is to present the setup for studying the formation of closed trapped surfaces in the head-on collision of two shock waves in dS space (this may be compared with the setup used to study the case without a cosmological constant in [11] and the case with the negative cosmological constant in [21, 25, 43]).

We briefly recall the results in [35, 39, 42] for the geometry of a shock wave propagating in the D-dimensional dS space-time (see appendix A). In terms of the dependent plane coordinates, $(u, v, \vec{x})$, $\vec{x} = (x^2, \dots, x^D)$, satisfying $-2uv + \vec{x}^2 = a^2$ (a is the cosmological radius), the line element of the shock wave space-time is

$$ds^2 = -2du\,dv + d\vec{x}^2 + F(\vec{x})\delta(u)du^2. \tag{2.1}$$

The shock wave shape function F is a fundamental solution of the equation

$$\left(\triangle_{\mathbb{S}^{D-2}} + \frac{D-2}{a^2}\right) F = -16\sqrt{2}\pi G_D \bar{p}\delta(\vec{n}, \vec{n}_0), \tag{2.2}$$

where $\triangle_{\mathbb{S}^{D-2}}$ is the Laplace-Beltrami operator on a $(D-2)$-dimensional sphere $\mathbb{S}^{D-2}$, $\vec{n} = \vec{x}/|\vec{x}|$, $\vec{n}_0$ is the location of the particle on the sphere, $\bar{p}$ is the energy of the shock wave, and G_D is the D-dimensional gravitational constant. The $\sqrt{2}$ in the right-hand side results from our choice of the coordinate system (if we impose $-du\,dv$ instead $-2du\,dv$ in the expression for the linear element, then it disappears). This metric is a solution of the Einstein equations for an energy-momentum tensor with the single nonvanishing component $T_{uu} \sim \bar{p}\delta(u)$. In the standard parameterization of the $(D-2)$-dimensional sphere by spherical angles $\vartheta_1, \dots, \vartheta_{D-2}$, the shock wave shape function corresponding to an ultra-relativistic point particle depends on only one spherical angle ϑ_{D-2}. The operator $\triangle_{\mathbb{S}^{D-2}}$ acts on $F = F(\vartheta_{D-2})$ as

$$\triangle_{\mathbb{S}^{D-2}} F = \frac{1}{a^2} \sin^{3-D}(\vartheta_{D-2}) \left(\frac{d}{d\vartheta_{D-2}} \sin^{D-3}(\vartheta_{D-2}) \frac{d}{d\vartheta_{D-2}} \right) F(\vartheta_{D-2}) \tag{2.3}$$

(see [37] regarding shock waves in dS/AdS with multipole structures).

For $D = 4$, we deal with

$$F(\xi) = 4\sqrt{2}p\left(-2 + \xi \ln\left(\frac{1+\xi}{1-\xi}\right)\right), \tag{2.4}$$

where $\xi = x^4/a = \cos\vartheta_2$ and $p = \bar{p}G_4$ is the rescaled energy.

We now consider a collision of two waves of the type described above. We suppose that in the region $\{u < 0\} \cup \{v < 0\}$, i.e., the part of the space-time before the collision, the metric is given by

$$ds^2 = -2du\,dv + d\vec{x}^2 + F(\xi, \xi_1)\delta(u)du^2 + F(\xi, \xi_2)\delta(v)dv^2, \tag{2.5}$$

where ξ_1 and ξ_2 are the locations of the two colliding particles (see [35] for the explicit formula for $F(\xi, \xi_i)$). In independent coordinates (see appendix C.1), the metric is

$$ds^2 = \frac{-2dw\,d\sigma + 2d\zeta\,d\bar{\zeta} + 2H_1(\zeta, \bar{\zeta})\,\delta(w)dw^2 + 2H_2(\zeta, \bar{\zeta})\,\delta(\sigma)d\sigma^2}{[1 - (w\sigma - \zeta\bar{\zeta})/2a^2]^2}, \tag{2.6}$$

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where $H_i(\zeta,\bar\zeta) = H(\zeta,\bar\zeta,\zeta_i,\bar\zeta_i)$, $\zeta_i = \zeta(\xi_i,\bar\xi_i)$, and

$$H(\zeta,\bar\zeta,0,0) = H(\zeta\bar\zeta) = \frac{1}{2}\left(1+\frac{1}{2a^2}\zeta\bar\zeta\right) F\left(\frac{1-\zeta\bar\zeta/2a^2}{1+\zeta\bar\zeta/2a^2}\right). \tag{2.7}$$

A rigorous analysis of the formation of black holes in collisions would require solving the Einstein equations in the interaction region $\{w>0,\sigma>0\}$ (see, e.g., [4, 47] and the references therein).

A sufficient condition for a black hole to form in the asymptotically flat case is the existence of a marginally closed trapped surface at the hypersurface $\{w\le 0,\sigma=0\}\cup\{w=0,\sigma\le 0\}$ [11, 25, 43–45]. We note that in non-asymptotically flat cases, there are no general theorems, but there is a common opinion that the existence of a marginally trapped surface can be used as an indication of black hole formation.

In the coordinates used in line element (2.6) null geodesics are discontinuous across the wave fronts, $w=0$ and $\sigma=0$ (see appendix C) . So using such coordinates to find this trapped surface equation is inconvenient and can be avoided by switching to a new coordinate system $(W,\Sigma,\Upsilon,\bar\Upsilon)$ in which the delta function terms are eliminated in the metric and the geodesics are continuous. Similar to the D'Eath and Payne coordinates [45], which are closely related to the explicit form of geodesics in the Minkowski space-time with the shock wave [33, 34], these coordinates are also closely related to the geodesics. This is a reason for us to study geodesics in the dS space-time with the shock wave (see appendix B for details and references).

In these coordinates the trapped surface that we seek has two parts, which are denoted here by $\mathcal{S}_1$ and $\mathcal{S}_2$ in the respective regions $\Sigma<0$ and $W<0$. They are defined in terms of the two functions $\Psi_1(\Upsilon,\bar\Upsilon)$ and $\Psi_2(\Upsilon,\bar\Upsilon)$ by

$$\mathcal{S}_1:\begin{cases}W=0,\\ \Sigma=-\Psi_1(\Upsilon,\bar\Upsilon),\end{cases}\qquad \mathcal{S}_2:\begin{cases}\Sigma=0,\\ W=-\Psi_2(\Upsilon,\bar\Upsilon),\end{cases} \tag{2.8}$$

with the additional boundary conditions at the shock wave intersection $\mathcal{C}\subset\{W=\Sigma=0\}$

$$\Psi_1(\Upsilon,\bar\Upsilon)\Big|_{\mathcal{C}}=0,\qquad \Psi_2(\Upsilon,\bar\Upsilon)\Big|_{\mathcal{C}}=0, \tag{2.9}$$

and

$$\partial_\Upsilon\Psi_1\partial_{\bar\Upsilon}\Psi_2\Big|_{\mathcal{C}}=1. \tag{2.10}$$

Because $\mathcal{S}_1$ and $\mathcal{S}_2$ are in the respective regions $W<0$ and $\Sigma<0$, we also have $\Psi_1(\Upsilon,\bar\Upsilon)>0$ and $\Psi_2(\Upsilon,\bar\Upsilon)>0$.

The two functions $\Psi_1(\Upsilon,\bar\Upsilon)$ and $\Psi_2(\Upsilon,\bar\Upsilon)$ must be determined by imposing the condition that the surface they define is marginally trapped [18, 46, 48], i.e., that the congruence of outgoing null geodesics orthogonal to the surface has zero expansion.

The zero convergence equation for the D=4 case reduces to the equation (see appendix D)

$$\left(\triangle_{\mathbb{S}^2}+\frac{2}{a^2}\right)\phi_{1,2}(\Upsilon,\bar\Upsilon)=0, \tag{2.11}$$

where

$$\triangle_{\mathbb{S}^2} = 2\left(1+\frac{\Upsilon\bar{\Upsilon}}{2a^2}\right)^2 \partial_\Upsilon \partial_{\bar{\Upsilon}} \tag{2.12}$$

is the Laplace-Beltrami operator in complex coordinates (see appendix D) and the functions $\phi_{1,2}$ are related to the functions $\Psi_{1,2}$ defining the two halves of the trapped surface and the shock wave shape functions $H_{1,2}$ (cf. similar equations in the AdS case [21, 25])

$$\phi_{1,2} = \frac{2\Psi_{1,2} - H_{1,2}}{1+\Upsilon\bar{\Upsilon}/2a^2}. \tag{2.13}$$

The next question to be discussed is whether a black hole forms as a result of the head-on collision of two waves of the type described above. Head-on collisions preserve rotational symmetry around the axis of motion of massless particles, the $O(2)$-symmetry in $D = 4$. Because a head-on collision is $O(2)$-symmetric, the functions $\Psi_1(\Upsilon,\bar{\Upsilon})$ and $\Psi_2(\Upsilon,\bar{\Upsilon})$ describing the trapped surface are identical and depend on only the parameter $\rho^2 = \Upsilon\bar{\Upsilon}$: $\Psi_1(\Upsilon,\bar{\Upsilon}) = \Psi_2(\Upsilon,\bar{\Upsilon}) = \Psi(\rho^2)$.

3 Marginally trapped surface in dS

3.1 Solution of the marginally trapped surface equation in dS_4

In the case $\phi_1(\Upsilon,\bar{\Upsilon}) = \phi_2(\Upsilon,\bar{\Upsilon}) = \phi(\rho^2)$, equation (2.11) is transformed into the ordinary differential equation

$$\left(1+\frac{\rho^2}{2a^2}\right)^2\left(\frac{\partial^2\phi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\phi}{\partial\rho}\right) + \frac{4\phi}{a^2} = 0. \tag{3.1}$$

The solutions of this equation are

$$\phi = \frac{A(\rho^2-2a^2) + B\big((\rho^2-2a^2)\ln\rho + 4a^2\big)}{\rho^2+2a^2}, \tag{3.2}$$

where A and B are constants. Because we are interested in a solution of a homogenous equation without singularities, we take $B = 0$ and obtain the expression for the trapped surface function in terms of the shape function H,

$$\Psi = \frac{1}{2}H(\rho) + \left(1+\frac{\rho^2}{2a^2}\right)\frac{A(\rho^2-2a^2)}{\rho^2+2a^2}, \tag{3.3}$$

and also in terms of the shape function F,

$$\Psi = \frac{1}{4}\left(1+\frac{\rho}{2a^2}\right)F(\rho) + \frac{1}{2a^2}A(\rho^2-2a^2) \tag{3.4}$$

or, more explicitly,

$$\Psi = \sqrt{2}p\left(1+\frac{\rho^2}{2a^2}\right)\left(-2+\frac{2a^2-\rho^2}{2a^2+\rho^2}\ln\left(\frac{2a^2}{\rho^2}\right)\right) + \frac{1}{2a^2}A(\rho^2-2a^2). \tag{3.5}$$

As mentioned above, the function Ψ must satisfy the following boundary conditions in the head-on case:

$$\Psi\Big|_{\mathcal{C}} = 0, \tag{3.6}$$

$$\partial_\Upsilon \Psi \partial_{\bar\Upsilon} \Psi\Big|_{\mathcal{C}} = 1. \tag{3.7}$$

It is obvious that the bound $\mathcal{C}$ is a circle $\rho = \rho_0 = \text{const}$. We hence have a system of two equations for the two constants A and ρ_0:

$$\sqrt{2}p\left(1+\frac{\rho_0^2}{2a^2}\right)\left(-2+\frac{2a^2-\rho_0^2}{2a^2+\rho_0^2}\ln\left(\frac{2a^2}{\rho_0^2}\right)\right)+\frac{1}{2a^2}A\cdot(\rho_0^2-2a^2)=0, \tag{3.8}$$

$$\frac{1}{4a^4\rho_0^2}\left(2\sqrt{2}pa^2+\rho_0^2\left(\sqrt{2}p-A\right)+\sqrt{2}p\rho_0^2\ln\left(\frac{2a^2}{\rho_0^2}\right)\right)^2=1. \tag{3.9}$$

Substituting A from (3.8) in expression (3.3), we obtain

$$\Psi(\rho)=\sqrt{2}p\left(4\frac{\rho^2-\rho_0^2}{\rho_0^2-2a^2}+\left(1-\frac{\rho^2}{2a^2}\right)\ln\left(\frac{\rho_0^2}{\rho^2}\right)\right). \tag{3.10}$$

Equation (3.9) defining ρ can be also written in terms of the initial function F defining the shock wave in the plane coordinates,

$$\frac{1}{4}\left(1+\frac{\rho_0^2}{2a^2}\right)F'(\rho_0)+\frac{\rho_0}{2a^2-\rho_0^2}F(\rho_0)+2=0. \tag{3.11}$$

Indeed, normalization condition (2.10) can be written in the form

$$\left(\frac{d\Psi}{d\rho}\right)^2=4 \quad \text{or} \quad \frac{d\Psi}{d\rho}\equiv\Psi'(\rho)=\pm 2. \tag{3.12}$$

We choose the minus sign. Because of relation (3.4) on $\mathcal{C}$, we have

$$\Psi'(\rho)\Big|_{\mathcal{C}}=\frac{1}{4}\left(1+\frac{\rho_0^2}{2a^2}\right)F'(\rho_0)+\frac{\rho_0}{2a^2-\rho_0^2}F(\rho_0), \tag{3.13}$$

and we obtain (2.10) in the form

$$\frac{1}{4}\left(1+\frac{\rho_0^2}{2a^2}\right)F'(\rho_0)+\frac{\rho_0}{2a^2-\rho_0^2}F(\rho_0)+2=0. \tag{3.14}$$

This equation is universal for an arbitrary dimension D. To connect with the AdS case, this equation can be rewritten in terms of the chordal coordinate related to ρ by

$$\rho=a\sqrt{\frac{2}{1-q}}. \tag{3.15}$$

After this change of variable, equation (3.11) becomes

$$F'(q_0)+\frac{2}{1-2q_0}F(q_0)+\frac{8a}{\sqrt{2q_0(1-q_0)}}=0. \tag{3.16}$$

We introduce the dimensionless parameter $x_0 = \rho_0/a$. Equation (3.9) becomes

$$f(x_0) = \sqrt{2}\frac{a}{p}, \tag{3.17}$$

where

$$f(x) \equiv \frac{1}{x}\frac{(2+x^2)^2}{2-x^2}. \tag{3.18}$$

We note that in the region $0 < x < \sqrt{2}$, the function $f(x)$ has the positive minimum

$$f'(x)|_{x=x_{\min}} = 0, \qquad x_{\min} = 2-\sqrt{2}. \tag{3.19}$$

Hence, in the case

$$\eta \equiv \frac{a}{p} < \frac{1}{\sqrt{2}} \cdot f(x_{\min}) = 4, \tag{3.20}$$

there are no solutions of (3.17) (see figure 1). Therefore, no solution of the trapped surface equation can be found below the critical value $\eta_c = 4$ of the parameter η. In terms of the rescaling energy $\bar{p} = p/G$, where G is the gravitational constant, this relation gives (1.1).

It is reasonable to consider two limit cases: where $x_0 \ll 1$ and $p < a$ (when the particle energy is low and/or the spacetime is weakly curved; it is natural to call this the low-energy limit) and where x_0 and η are equal to the critical values (we call this the high-energy critical limit).

From equation (3.17) in the region $x_0 \ll 1$, we obtain

$$x_0 \approx \frac{\sqrt{2}p}{a} \quad \text{and} \quad \rho_0 \approx \sqrt{2}p. \tag{3.21}$$

And in the critical limit, we obtain

$$p = \frac{a}{4}, \qquad \rho_0 = (2-\sqrt{2})a = (8-4\sqrt{2})p. \tag{3.22}$$

3.2 Higher-dimension cases

Similar calculations can be performed in higher dimensions. Equation (3.14) has a universal form and is independent of the dimension D. The variable ρ for an arbitrary dimension is related to $\xi \equiv Z_D/a$ as

$$\xi = \frac{1-\rho^2/2a^2}{1+\rho^2/2a^2}. \tag{3.23}$$

As examples, we consider the cases $D = 5$ and $D = 6$. The shape functions have the forms

$$F_5(\xi) = \frac{3\sqrt{2}\pi p_5}{a}\frac{2\xi^2-1}{\sqrt{1-\xi^2}} \tag{3.24}$$

$$F_6(\xi) = \frac{8\sqrt{2}p_6}{a^2}\left(\xi \ln\left(\frac{1+\xi}{1-\xi}\right) + \frac{2(3\xi^2-2)}{3(1-\xi^2)}\right), \tag{3.25}$$

where $\xi = Z_D/a$ and $p_D = \bar{p}G_D$. For these cases, equation (3.14) reduces to

$$f_D(x_0) = C_D\frac{a^{D-3}}{p_D}, \tag{3.26}$$

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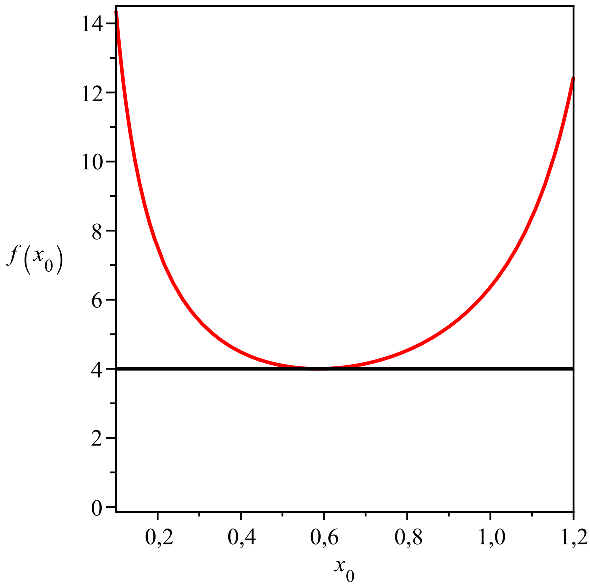

Figure 1. The critical value of the parameter $\eta = a/p$ corresponds to the minimum value of the function f (red line).

where C_D is a D-dependent constant (in particular, $C_3 = 2/\pi$, $C_5 = 32/3\pi$, and $C_6 = 6\sqrt{2}$) and the functions $f_D(x)$ are given by

$$f_D(x) = \frac{(2+x^2)^{D-2}}{x^{D-3}(2-x^2)}. \tag{3.27}$$

These functions have positive minima at the points

$$x_{min,D} = \frac{\sqrt{2(-1+D-2\sqrt{D-2})}}{\sqrt{D-3}}, \qquad D > 3, \tag{3.28}$$

$$x_{min,3} = 0, \qquad D = 3, \tag{3.29}$$

and the value of the functions $f_D(x)$ at these points, $f_{0,D} \equiv f_D(x_{min,D})$, give the critical values of the parameter $\bar{p}$ below which trapped surfaces can be formed,

$$\bar{p} < \bar{p}_{cr,D}, \tag{3.30}$$

$$\bar{p}_{cr,D} = \frac{a^{D-3}}{G_D}\frac{C_D}{f_{0,D}}. \tag{3.31}$$

We note that this critical effect does not necessarily exclude the trapped surface in the region of interacting shock waves $W > 0$, $\Sigma > 0$.

We can also interpret formula (3.31) to mean that for a fixed value of $\bar{p}$, there is a critical value of the de Sitter radius a_{cr},

$$a_{cr} \equiv \left(\bar{p} G_D \frac{f_{0,D}}{C_D}\right)^{1/D-3}, \quad D > 3, \tag{3.32}$$

only above which can trapped surfaces be formed. For $D = 3$, the critical energy is independent of a.

In particular, for $D = 5$ and $D = 6$, we have

$$x_{min,5} = -1 + \sqrt{3}, \tag{3.33}$$

$$x_{min,6} = \frac{\sqrt{6}}{3} \tag{3.34}$$

and

$$f_5(x_{5,min}) = 12\sqrt{3}, \tag{3.35}$$

$$f_6(x_{6,min}) = \sqrt{6}\frac{2^8}{3^2}. \tag{3.36}$$

Hence, the boundary problem can be solved in $D = 5$ and $D = 6$ only if the conditions

$$\frac{a^2}{p_5} > \frac{9}{8}\pi\sqrt{3}, \tag{3.37}$$

$$\frac{a^3}{p_6} > \frac{128}{27}\sqrt{3} \tag{3.38}$$

are satisfied. We thus have the same critical effect as in the four-dimensional dS space-time. The trapped surface can be formed only if the energy of colliding particles is not very high. Assuming that $x_0 \ll 1$, we obtain

$$\rho_0 \approx 2\left(\frac{\bar{p}G_D}{C_D}\right)^{1/(D-3)} \tag{3.39}$$

from (3.26).

3.3 Area of the marginally trapped surface below the critical point

Knowing ρ_0, we can calculate the area of the trapped surface:

$$\mathcal{A}_{\rm trap} = 2\int\limits_{\rho<\rho_0}\sqrt{\det g_{\alpha\beta}}\,dV, \tag{3.40}$$

where $g_{\alpha\beta}$ is the induced metric on the trapped surface and dV is an elementary volume in the $(D-2)$-dimensional flat space. Because the corresponding induced metric in the four- and five-dimensional space-times has the form (the form is similar in higher dimensions)

$$g_{\alpha\beta} = \frac{1}{\mathcal{N}}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad g_{\alpha\beta} = \frac{1}{\mathcal{N}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{3.41}$$

the explicit expression for the area of the trapped surface is

$$\mathcal{A}_{\rm trap} = 2\cdot{\rm Vol}\,S^{D-3}\int\limits_0^{\rho_0}\frac{2^{\frac{D-2}{2}}\rho^{D-3}}{\mathcal{N}^{(D-2)/2}}d\rho = 2\cdot{\rm Vol}\,S^{D-3}\int\limits_0^{\rho_0}\frac{2^{\frac{D-2}{2}}\rho^{D-3}}{(1+\rho^2/2a^2)^{D-2}}d\rho. \tag{3.42}$$

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In particular,

$$\mathcal{A}^4 = 8\pi \frac{a^2\rho_0^2}{2a^2+\rho_0^2}, \tag{3.43}$$

$$\mathcal{A}^5 = 4\sqrt{2}\pi a^3 \left(\sqrt{2}\arctan\left(\frac{\sqrt{2}}{2}\frac{\rho_0}{a}\right) + \frac{2a\rho_0(\rho_0^2-2a^2)}{(2a^2+\rho_0^2)^2}\right). \tag{3.44}$$

We have

$$\mathcal{A}^4_{\mathrm{LE}} \approx 8\pi \frac{a^2p^2}{a^2+p^2} \approx 8\pi p^2\left(1-\frac{p^2}{a^2}\right), \tag{3.45}$$

$$\mathcal{A}^5_{LE} \approx \frac{16\sqrt{2}\pi}{3}\rho_0^3 \approx \sqrt{\frac{3}{2\pi}p_5^3} \tag{3.46}$$

in the low-energy limit and

$$\mathcal{A}^4_{C\mathrm{r}} = (4-2\sqrt{2})\pi a^2, \tag{3.47}$$

$$\mathcal{A}^5_{\mathrm{Cr}} = \frac{8\sqrt{2}\pi}{(3-\sqrt{3})^2}(\sqrt{3}-2)\left(1-3\sqrt{2}\arctan\frac{\sqrt{2}(\sqrt{3}-1)}{2}\right)a^3 \tag{3.48}$$

at the critical point.

4 Concluding remarks

We have studied the formation of marginally trapped surfaces in head-on collisions of two shock waves in the dS space-time for $D \leq 3$. For $D \leq 4$, we found the critical value of of the shock wave energy dependent on the dS radius, above which the trapped surface equation has no solution. This critical behavior is similar to that found in [21] and is also reminiscent of the behavior encountered in numerical simulations of gravitational collapse [23, 24]. For $D = 3$, the critical energy above which there is no trapped surface is independent of the cosmological constant.

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A Geometric view of shock waves in plane coordinates

A single shock wave in the D-dimensional dS space is shown in figure 2.A. The dS space is represented as a hyperboloid embedded into the $(D+1)$-dimensional Minkowski space-time. The presented shock wave is located on the intersection of the hyperboloid and the plane $x^0 - x^1 = 0$,

$$u = \frac{x^0+x^1}{\sqrt{2}}, \qquad v = \frac{x^0-x^1}{\sqrt{2}}.$$

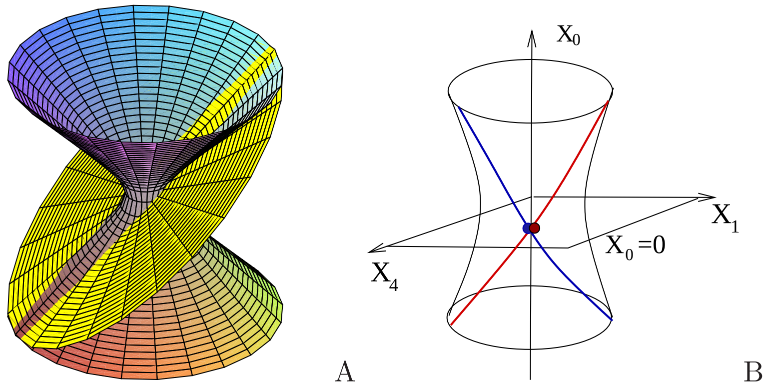

Figure 2. A. A single shock wave in the dS space can be represented as the intersection of the hyperboloid and the plane $x^0 - x^1 = 0$ (x^2 and x^3 are suppressed). B. Two shock waves in the dS space. A collision of two shock waves occurs at $x^0 = 0$ and corresponds to the collision of red and yellow balls.

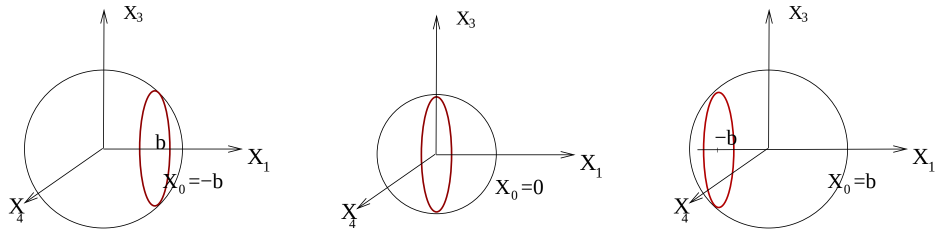

Figure 3. A shock wave in the dS space at different instants of "x^0 time."

The coordinates x_2 and x_3 are suppressed in this figure.

Two shock waves colliding at $u = v = 0$ are shown in figure 2.B.

We can also make an animation and draw the position of the single shock wave at discrete instants. In figure 3, we see that the shock wave is a nonexpanding one.

B Solution of the geodesics equation

B.1 The σ-model (n-field) approach

In this section, we derive and solve the null-geodesic equations in the dS space-time with a nonexpanding shock-wave. We note that applying the embedding theorems [40] requires a delicate analysis because of the nonsmoothness of the metric with the shock wave.

We use an analytical approach similar to the σ-model (n-field) approach and start from the Lagrangian

$$\mathcal{L} = \int d\tau \left[\frac{dx^M(\tau)}{d\tau} G_{MN}(x(\tau)) \frac{dx^N(\tau)}{d\tau} - \lambda \left(x^M(\tau) g_{MN} x^N(\tau) - a^2 \right) \right], \tag{B.1}$$

where $M, N = 0, \ldots, D$, g_{MN} is the metric defined the hyperboloid, and $G_{MN}(x(\tau))$ is the metric deformed by the shock wave,

$$G_{MN} = g_{MN} + F\delta(u) du^2. \tag{B.2}$$

The Euler equations for this Lagrangian are

$$G_{MN}\frac{d^2x^N(\tau)}{d\tau^2}+G_{MN}\Gamma^N_{KL}\frac{dx^K(\tau)}{d\tau}\frac{dx^L(\tau)}{d\tau}+\lambda g_{MN}x^N(\tau)=0, \tag{B.3}$$

$$x^M(\tau)g_{MN}x^N(\tau)-a^2=0. \tag{B.4}$$

Using $u\delta(u)=0$, we obtain

$$G^{MN}g_{NK}x^K=x^M, \tag{B.5}$$

and equations (B.3) and (B.4) reduce to

$$\ddot{x}^N+\Gamma^N_{KL}\dot{x}^K\dot{x}^L+\lambda\,x^N=0,\qquad \lambda=-\frac{1}{a^2}(x,\ddot{x}+\Gamma_{KL}\dot{x}^K\dot{x}^L)_G. \tag{B.6}$$

Here and hereafter, $(x,y)_G=G_{MN}x^My^N$. The nonvanishing components of the connection Γ^M_{NK} are

$$\Gamma^v_{uu}=-\frac{1}{2}F\delta'(u),\qquad \Gamma^v_{ui}=-\frac{1}{2}F_{,i}\delta(u),\qquad \Gamma^i_{uu}=-\frac{1}{2}F_{,i}\delta(u). \tag{B.7}$$

Taking $(x,x)_g=a^2$ and $G_{\mu\nu}\dot{x}^\mu\dot{x}^\nu=0$ into account, we obtain

$$\lambda=\frac{1}{2a^2}(-F+x^iF_{,i})\delta(u)\dot{u}^2. \tag{B.8}$$

Substituting this expression in (B.6) and taking (B.7) into account, we obtain

$$\ddot{u}=-\frac{1}{2a^2}(-F+x^iF_{,i})\delta(u)\dot{u}^2u, \tag{B.9}$$

$$\ddot{v}-\frac{1}{F}\delta'(u)\dot{u}^2-F_{,i}\delta(u)\dot{u}\dot{x}^i=-\frac{1}{2a^2}(-F+x^iF_{,i})\delta(u)\dot{u}^2v, \tag{B.10}$$

$$\ddot{x}^i-\frac{1}{2}F_{,i}\delta(u)\dot{u}^2=-\frac{1}{2a^2}(-F+x^iF_{,i})\delta(u)\dot{u}^2x^i. \tag{B.11}$$

Noting that the right-hand side of (B.9) vanishes, we obtain $\ddot{u}=0$. Taking $\tau=u$, we obtain

$$\ddot{v}-\frac{1}{2}F\delta'(u)-F_{,i}\delta(u)\dot{x}^i=-\frac{1}{2a^2}(-F+x^iF_{,i})v\delta(u), \tag{B.12}$$

$$\ddot{x}^i-\frac{1}{2}F_{,i}\delta(u)=-\frac{1}{2a^2}(-F+x^jF_{,j})x^i\delta(u). \tag{B.13}$$

We now use the ansatz (by analogy with the case of the shock wave in the Minkowski space-time [33, 34])

$$v=v_0+v_1u+Q(x^j_0)\theta(u)+R(x^j_0)\theta(u)u, \tag{B.14}$$

$$x^i=x_{i0}+x_{i1}u+x_{if}\theta(u)+S_i(x^j_0)\theta(U)U, \tag{B.15}$$

$$\dot{v}=v_1+Q(x^j_0)\delta(u)+R(x^j_0)\theta(u), \tag{B.16}$$

$$\dot{x}^i=x_{i1}+D_i(x^j_0)\delta(u)+S_i(x^j_0)\theta(u), \tag{B.17}$$

$$\ddot{v}=Q(x^j_0)\delta'(u)+R(x^j_0)\delta(u), \tag{B.18}$$

$$\ddot{x}^i=D_i(x^j_0)\delta'(u)+S_i(x^j_0)\delta(u). \tag{B.19}$$

Along the geodesics, we have the identity

$$F(x^i)\delta'(u) = F(x_0^i)\delta'(u) - F_{,i}(x_0^i)\dot{x}^i\delta(u) \tag{B.20}$$

(a similar identity for the flat space-time case was used in [34]). Transforming our anzatz in order to cancel $\delta^2(u)$, we now obtain

$$\begin{aligned} v &= v_0 + v_1 u + Q(x_0^j)\theta(u) + R(x_0^j)\theta(u)u, \\ x^i &= x_0^i + x_1^i u + S_i\theta(u)u \end{aligned} \tag{B.21}$$

with the bounds

$$x_{i0}^2 = a^2, \tag{B.22}$$

$$v_0 = x_0^i x_1^i, \tag{B.23}$$

$$v_1 = \frac{1}{2}x_1^{i2}. \tag{B.24}$$

And hence

$$Q = \frac{1}{2}F, \tag{B.25}$$

$$R = \frac{1}{2}F_{,i}x_1^i + \frac{1}{2a^2}(F - x_0^i F_{,i})v_0 + \frac{1}{8}F_{,i}^2 + \frac{1}{8a^2}(F^2 - (x_0^i F_{,i})^2), \tag{B.26}$$

$$S_i = \frac{1}{2}F_{,i} + \frac{1}{2a^2}(F - x_0^j F_{,j})x_0^i. \tag{B.27}$$

We can take $x_1^i = 0$ for simplicity. This gives $v_0 = v_1 = 0$, and we have

$$\begin{aligned} Q &= \frac{1}{2}F \\ R &= \frac{1}{8}F_{,i}^2 + \frac{1}{8a^2}(F^2 - (x_0^i F_{,i})^2), \\ S_i &= \frac{1}{2}F_{,i} + \frac{1}{2a^2}(F - x_0^j F_{,j})x_0^i. \end{aligned} \tag{B.28}$$

B.2 Focusing of geodesics

Before deriving the explicit expression for the trapped surface, we study the structure of geodesic beams in terms of the dependent plane coordinates. We consider points that are initially on the surface $x_{20}^2 + x_{30}^2 + x_{40}^2 = a^2$ and change to the angular coordinates:

$$\begin{aligned} &x_{20} = a\cdot\sin\phi\cdot\sin\theta, \qquad x_{30} = a\cdot\sin\phi\cdot\cos\theta, \qquad x_{40} = a\cdot\cos\phi, \\ &F = 4\sqrt{2}p\left(-2 + \frac{x_{40}}{a}\ln\left(\frac{a+x_{40}}{a-x_{40}}\right)\right) = 4\sqrt{2}p\left(-2 + \cos\phi\ln\left(\frac{1+\cos\phi}{1-\cos\phi}\right)\right). \end{aligned}$$

It is obvious that the term $S_i(x_0^j)\theta(u)u$ in formula (B.17) leads to the refraction of the geodesics. The refraction coefficients are $S_i(x_0^j)$. For the refraction coefficients in the angular parameterization, we have

$$S_2 = -\sqrt{2}\,\frac{p}{a}\frac{\sin\theta}{\sin\phi}, \qquad S_3 = -\sqrt{2}\,\frac{p}{a}\frac{\cos\theta}{\sin\phi}, \qquad S_4 = \frac{\sqrt{2}}{2}\frac{p}{a}\ln\left(\frac{1+\cos\phi}{1-\cos\phi}\right). \tag{B.29}$$

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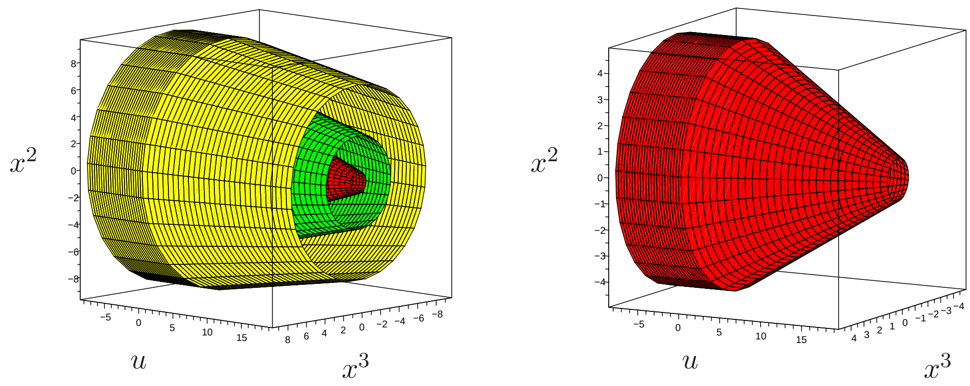

Figure 4. Focusing of geodesics: in the dependent coordinates, the focal length changes along with the initial conditions.

The coordinates x_2 and x_3 correspond to the physical coordinates related to the size of the beam. Hence, we can easily find the value of the affine parameter u, which determines the focal point of the beam (the point $x_2 = x_3 = 0$):

$$u = \frac{a^2}{\sqrt{2}p} \cdot \sin^2 \phi.$$

The "focal length" is different for each value of the parameter ϕ, i.e., for each circular ring of the beam cross section. This statement is illustrated in figure 4. Of course, this only roughly explains why the trapped surface can exist. To find a physical meaning for this consideration, we must change coordinates to independent ones.

C The shock wave in the independent coordinates

C.1 From plane coordinates to independent coordinates

To study the structure of the space-time in terms of the independent four-dimensional coordinates, it is convenient to use the complex conformal flat coordinates

$$w = \frac{2au}{x^4 + a}, \qquad \sigma = \frac{2av}{x^4 + a}, \qquad \zeta = \frac{\sqrt{2}a}{x^4 + a}(x^2 + ix^3). \tag{C.1}$$

In these coordinates, the shock wave metric is

$$ds^2 = \frac{-2dw\, d\sigma + 2d\zeta\, d\bar{\zeta} + 2H(\zeta, \bar{\zeta})\delta(w)\, dw^2}{[1 - (w\sigma - \zeta\bar{\zeta})/2a^2]^2}, \tag{C.2}$$

where

$$H(\zeta, \bar{\zeta}) = \frac{1}{2}\left(1 + \frac{1}{2a^2}\zeta\bar{\zeta}\right) F\left(\frac{1 - \zeta\bar{\zeta}/2a^2}{1 + \zeta\bar{\zeta}/2a^2}\right) \tag{C.3}$$

and F is given by (2.4). Hence,

$$H(\zeta, \bar{\zeta}) = 2\sqrt{2}p\left(1 + \frac{1}{2a^2}\zeta\bar{\zeta}\right)\left(-2 + \frac{1 - \zeta\bar{\zeta}/2a^2}{1 + \zeta\bar{\zeta}/2a^2} \ln\left(\frac{2a^2}{\zeta\bar{\zeta}}\right)\right). \tag{C.4}$$

C.2 An analogue of the D'Eath and Payne coordinates (from the coordinates w, σ, ζ to the coordinates W, Σ, Υ)

To eliminate $\delta(u)$ from the metric, we use a coordinate change analogous to the change introduced in [45],

$$\begin{aligned} w &= W, \\ \sigma &= \Sigma + H(\Upsilon, \bar{\Upsilon})\theta(W) + W\theta(W)H_\Upsilon H_{\bar{\Upsilon}}, \\ \zeta &= \Upsilon + W\theta(W)H_{\bar{\Upsilon}}, \end{aligned} \tag{C.5}$$

where $H_\Upsilon = \partial_\Upsilon H(\Upsilon, \bar{\Upsilon})$. In these coordinates, we obtain the metric

$$ds^2 = \frac{-2dW\, d\Sigma + 2|d\Upsilon + W\theta(W)(H_{\Upsilon\bar{\Upsilon}}d\Upsilon + H_{\bar{\Upsilon}\bar{\Upsilon}}d\bar{\Upsilon})|^2}{[1 - (W\Sigma - \Upsilon\bar{\Upsilon} + W\theta(W)G)/2a^2]^2}, \tag{C.6}$$

where $G = H - \Upsilon H_\Upsilon - \bar{\Upsilon} H_{\bar{\Upsilon}}$ and $H(\Upsilon, \bar{\Upsilon})$ depends on $\Upsilon, \bar{\Upsilon}$ as $H(\zeta, \bar{\zeta})$ given by (C.3) depends on $\zeta, \bar{\zeta}$.

C.3 Geodesics in terms of the independent complex conformal flat coordinates

We have the expressions for geodesics in terms of the dependent coordinates. Using coordinate change (C.1), we obtain an expression for geodesics in terms of the independent coordinates. In the first order of the parameter u, we obtain

$$\begin{aligned} w(u) &= w_1 u + \dots, \\ \sigma(u) &= \sigma_0 + \sigma_1 u + \dots \equiv \sigma_{0c} + \sigma_{0\theta}\theta(u) + (\sigma_{1c} + \sigma_{1\theta}\theta(u))u + \dots, \\ \zeta(u) &= \zeta_0 + \zeta_1 u + \dots \equiv \zeta_{0c} + \zeta_{0\theta}\theta(u) + (\zeta_{1c} + \zeta_{1\theta}\theta(u))u + \dots, \end{aligned} \tag{C.7}$$

where

$$w_1 = \frac{2}{1 + x_0^4/a}, \tag{C.8}$$

$$\sigma_{0c} = \frac{2v_0}{1 + x_0^4/a}, \tag{C.9}$$

$$\sigma_{0\theta} = \frac{2Q(x_0^i)}{1 + x_0^4/a}, \tag{C.10}$$

$$\sigma_{1c} = 2\left(\frac{v_1}{1 + x_0^4/a} - \frac{x_1^4}{a}\frac{v_0}{(1 + x_0^4/a)^2}\right), \tag{C.11}$$

$$\sigma_{1\theta} = 2\left(\frac{R(x_0^i)}{1 + x_0^4/a} - \frac{Q(x_0^i)x_1^4}{a(1 + x_0^4/a)^2} - \frac{QS^4(x_0^i)}{a(1 + x_0^4/a)^2} - \frac{S^4(x_0^4)v_0}{a(1 + x_0^4 a)^2}\right), \tag{C.12}$$

$$\zeta_{0c} = \frac{\sqrt{2}z_0}{1 + x_0^4/a}, \tag{C.13}$$

$$\zeta_{0\theta} = 0, \tag{C.14}$$

$$\zeta_{1c} = \frac{\sqrt{2}z_1}{1 + x_0^4/a} - \frac{x_1^4}{a}\frac{\sqrt{2}z_0}{(1 + x_0^4/a)^2}, \tag{C.15}$$

$$\zeta_{1\theta} = \frac{\sqrt{2}\mathcal{S}}{1 + x_0^4/a} - \frac{S^4}{a}\frac{\sqrt{2}z_0}{(1 + x_0^4/a)^2}, \tag{C.16}$$

where the complex variables $\mathcal{S}$, z_0, z_1 are related to S^i, x_0^i, x_1^i by

$$\mathcal{S} = S^2 + iS^3, \tag{C.17}$$

$$z_0 = x_0^2 + ix_0^3, \qquad z_1 = x_1^2 + ix_1^3. \tag{C.18}$$

We see that there is a discontinuity only for the σ variable.

C.4 Geodesics in terms of the independent smooth coordinates

Using relations (C.5) between the initial independent coordinates and the smooth independent coordinates, we obtain the expression for geodesics in terms of the smooth independent coordinates up to the second order:

$$\Sigma(w) = \big(\Sigma_{0c} + \Sigma_{0\theta}\theta(w)\big) + \big(\Sigma_{1c} + \Sigma_{1\theta}\theta(w)\big)w + \Sigma_2\frac{w^2}{2} + \ldots, \tag{C.19}$$

$$\Upsilon(w) = \Upsilon_0 + \big(\Upsilon_{1c} + \Upsilon_{1\theta}\theta(w)\big)w + \Upsilon_2\frac{w^2}{2} + \ldots, \tag{C.20}$$

$$\bar{\Upsilon}(w) = \bar{\Upsilon}_0 + \big(\bar{\Upsilon}_{1c} + \bar{\Upsilon}_{1\theta}\theta(w)\big)w + \bar{\Upsilon}_2\frac{w^2}{2} + \ldots, \tag{C.21}$$

where

$$\Sigma_{0c} = \sigma_0, \tag{C.22}$$

$$\Sigma_{0\theta} = 0, \tag{C.23}$$

$$\Sigma_{1c} = \sigma_{1c}, \tag{C.24}$$

$$\Sigma_{1\theta} = \sigma_{1\theta} - H_{\Upsilon}(\Upsilon_0, \bar{\Upsilon}_0)\Upsilon_1 - H_{\bar{\Upsilon}}(\Upsilon_0, \bar{\Upsilon}_0)\bar{\Upsilon}_1 - w_1 H_{\Upsilon}(\Upsilon_0, \bar{\Upsilon}_0)H_{\bar{\Upsilon}}(\Upsilon_0, \bar{\Upsilon}_0), \tag{C.25}$$

$$\Upsilon_0 = \zeta_{0c}, \tag{C.26}$$

$$Y_{1c} = \zeta_{1c}, \tag{C.27}$$

$$Y_{1\theta} = \zeta_{1\theta} - w_1 H_{\bar{\Upsilon}}(\Upsilon_0, \bar{\Upsilon}_0). \tag{C.28}$$

D Trapped surface equation

In the case of two shock waves, we deal with

$$\begin{aligned} ds^2 &= 2g_{W\Sigma}dW d\Sigma + g_{\Upsilon\Upsilon}d\Upsilon d\Upsilon + 2g_{\Upsilon\bar{\Upsilon}}d\Upsilon d\bar{\Upsilon} + g_{\bar{\Upsilon}\bar{\Upsilon}}d\bar{\Upsilon}d\bar{\Upsilon} = \\ &= \frac{-2dW\,d\Sigma + 2|d\Upsilon + \big(\Sigma\theta(\Sigma) + W\theta(W)\big)(H_{\Upsilon\bar{\Upsilon}}d\Upsilon + H_{\bar{\Upsilon}\bar{\Upsilon}}d\bar{\Upsilon})|^2}{[1 - (W\Sigma - \Upsilon\bar{\Upsilon} + (\Sigma\theta(\Sigma) + W\theta(W))G/2a^2)]^2}. \end{aligned} \tag{D.1}$$

Because of the notation symmetry ($W \leftrightarrow \Sigma$), we may analyze only one part of the trapped surface.

The null geodesics passing through the surface

$$\Sigma = \Psi(\Upsilon, \bar{\Upsilon}), \qquad W = 0 \tag{D.2}$$

can be specified by the tangent vectors (see Subsections C.3 and C.4):

$$\xi^W = w_1, \qquad \xi^\Sigma = \sigma_1, \qquad \xi^\Upsilon = \zeta_1, \qquad \xi^{\bar{\Upsilon}} = \bar{\zeta}_1, \tag{D.3}$$

where

$$\sigma_1 = -\frac{1}{2g_{W\Sigma}w_1}(g_{\Upsilon\Upsilon}\zeta_1^2 + 2g_{\Upsilon\bar{\Upsilon}}\zeta_1\bar{\zeta}_1 + g_{\bar{\Upsilon}\bar{\Upsilon}}\bar{\zeta}_1^2). \tag{D.4}$$

We also suppose that

$$(\xi, K_a) = 0, \qquad a = \zeta, \bar{\zeta}, \tag{D.5}$$

for

$$K_a^M = (0, -\partial_a\Psi, \delta_a^b) \tag{D.6}$$

(our calculations are very close to those in the review section in [28]). This gives

$$-g_{W\Sigma}\partial_a\Psi w_1 + g_{ab}\zeta^b = 0. \tag{D.7}$$

From (D.7), we have

$$\zeta^a = w_1 g^{ab} g_{W\Sigma}\partial_b\Psi, \tag{D.8}$$

where g^{ab} is the inverse metric for g_{ab}. Therefore, we have

$$\xi^W = w_1, \tag{D.9}$$

$$\xi^\Sigma = \sigma_1 = -\frac{w_1 g_{W\Sigma}}{2\det(g^{ab})}(g_{\bar{\Upsilon}\bar{\Upsilon}}\partial_{\bar{\Upsilon}}\Psi\partial_{\bar{\Upsilon}}\Psi - 2g_{\Upsilon\bar{\Upsilon}}\partial_\Upsilon\Psi\partial_{\bar{\Upsilon}}\Psi + g_{\Upsilon\Upsilon}\partial_{\bar{\Upsilon}}\Psi\partial_{\bar{\Upsilon}}\Psi), \tag{D.10}$$

$$\xi^\Upsilon = \zeta_1 = \frac{w_1 g_{W\Sigma}}{\det(g^{ab})}(g_{\bar{\Upsilon}\bar{\Upsilon}}\partial_\Upsilon\Psi - g_{\Upsilon\bar{\Upsilon}}\partial_{\bar{\Upsilon}}\Psi), \tag{D.11}$$

$$\xi^{\bar{\Upsilon}} = \bar{\zeta}_1 = \frac{w_1 g_{W\Sigma}}{\det(g^{ab})}(g_{\Upsilon\Upsilon}\partial_{\bar{\Upsilon}}\Psi - g_{\Upsilon\bar{\Upsilon}}\partial_\Upsilon\Psi). \tag{D.12}$$

For $\xi_M = g_{MN}\xi^N$ at the point $W = 0$, we have

$$\xi_M = \left(-\frac{\partial_\Upsilon\Psi\partial_{\bar{\Upsilon}}\Psi}{1+\Upsilon\bar{\Upsilon}/2a^2}, -\frac{1}{1+\Upsilon\bar{\Upsilon}/2a^2}, -\frac{\partial_\Upsilon\Psi}{1+\Upsilon\bar{\Upsilon}/2a^2}, -\frac{\partial_{\bar{\Upsilon}}\Psi}{1+\Upsilon\bar{\Upsilon}/2a^2}\right). \tag{D.13}$$

For the convergence at the surface $W = 0, \Sigma = -\Psi(\zeta, \bar{\zeta})$, we obtain

$$\theta = h^{MN}\nabla_N\xi_M, \tag{D.14}$$

where

$$h^{MN} = K_\alpha^M g^{\alpha\beta} K_\beta^N \tag{D.15}$$

and $g^{\alpha\beta}$ is the inverse of the metric induced on the trapped surface,

$$g_{\alpha\beta} = K_\alpha^M g_{MN} K_\beta^N = \begin{pmatrix} g_{\Upsilon\Upsilon} & g_{\Upsilon\bar{\Upsilon}} \\ g_{\bar{\Upsilon}\Upsilon} & g_{\bar{\Upsilon}\bar{\Upsilon}} \end{pmatrix}. \tag{D.16}$$

The components of the connection for metric (D.1) in the coordinates $X^M = (W, \Sigma, \Upsilon, \bar{\Upsilon})$, $M, N = 0, 1, 2, 3$, are

$$\Gamma^1_{11} = -\frac{\partial_\Sigma\mathcal{N}}{\mathcal{N}}, \qquad \Gamma^1_{12} = -\frac{\partial_\Upsilon\mathcal{N}}{2\mathcal{N}}, \qquad \Gamma^2_{12} = -\frac{\partial_\Sigma\mathcal{N}}{2\mathcal{N}}, \tag{D.17}$$

$$\Gamma^1_{22} = \frac{H_{\Upsilon\Upsilon}}{2}, \qquad \Gamma^2_{22} = -\frac{\partial_\Upsilon\mathcal{N}}{\mathcal{N}}, \qquad \Gamma^1_{13} = -\frac{\partial_{\bar{\Upsilon}}\mathcal{N}}{2\mathcal{N}}, \tag{D.18}$$

$$\Gamma^3_{13} = -\frac{\partial_\Sigma \mathcal{N}}{2\mathcal{N}}, \qquad \Gamma^1_{23} = -\frac{1}{2\mathcal{N}}(\partial_W \mathcal{N} - H_{\Upsilon\bar{\Upsilon}}\mathcal{N}), \qquad \Gamma^0_{23} = -\frac{\partial_\Sigma \mathcal{N}}{2\mathcal{N}}, \tag{D.19}$$

$$\Gamma^1_{33} = \frac{H_{\bar{\Upsilon}\bar{\Upsilon}}}{2}, \qquad \Gamma^3_{33} = -\frac{\partial_{\bar{\Upsilon}}\mathcal{N}}{\mathcal{N}}, \tag{D.20}$$

where

$$\mathcal{N} = \left[1 - \left(W\Sigma - \Upsilon\bar{\Upsilon} + (\Sigma\theta(\Sigma) + W\theta(W))G\right)/2a^2\right]^2. \tag{D.21}$$

At the point $W = 0$, the tensor h^{MN} can be represented as

$$h^{MN} = \frac{1}{\det(g_{\alpha\beta})}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2\partial_\Upsilon\Psi\partial_{\bar{\Upsilon}}\Psi & \partial_{\bar{\Upsilon}}\Psi & \partial_\Upsilon\Psi \\ 0 & \partial_{\bar{\Upsilon}}\Psi & 0 & -1 \\ 0 & \partial_\Upsilon\Psi & -1 & 0 \end{pmatrix}. \tag{D.22}$$

Finally, the equation for the trapped surface can be found by direct calculations:

$$\left(\left(1 + \frac{\Upsilon\bar{\Upsilon}}{2a^2}\right)^2 \partial_{\Upsilon\bar{\Upsilon}} + \frac{1}{a^2}\right)\frac{2\Psi - H}{1 + \Upsilon\bar{\Upsilon}/2a^2} = 0. \tag{D.23}$$

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